Aufgabe 1) Creation and annihilation operators

Proof the following relations for the fermionic creation $\hat{a}^\dagger_i$ and annihilation $\hat{a}_i$ operators and the particle number operator $\hat{n}_i = \hat{a}^\dagger_i \hat{a}_i$:

- $\hat{n}_i^2 = \hat{n}_i$
- $\hat{a}_i \hat{n}_i = \hat{a}_i$
- $\hat{a}^\dagger_i \hat{n}_i = 0$
- $\hat{n}_i \hat{a}_i = 0$
- $\hat{n}_i \hat{a}^\dagger_i = \hat{a}^\dagger_i$

Aufgabe 2) Matrix elements

Calculate the matrix element of bosonic and fermionic operators over the vacuum state $|0\rangle$

$$< 0 | \hat{a}_i \hat{a}_j \hat{a}_k^\dagger \hat{a}_m^\dagger | 0 >$$

Aufgabe 3) Coulomb Interaction

Using the general representation of the two-particle interaction in the field operators:

$$H_{int} = \frac{1}{2} \int d^3r d^3r' \hat{\Psi}(\vec{r})^\dagger \hat{\Psi}(\vec{r}')^\dagger U(\vec{r} - \vec{r}') \hat{\Psi}(\vec{r}') \hat{\Psi}(\vec{r})$$

show that the Coulomb interaction $U = \frac{1}{(|\vec{r} - \vec{r}'|)}$ (in a.u.) for spinless fermions has the following form in impulses $\vec{k}$-space:

$$H_{int} = \frac{1}{2V} \sum_{\vec{k}, \vec{k}', \vec{q}} U_q \hat{a}^\dagger_{\vec{k} + \vec{q}} \hat{a}^\dagger_{\vec{k}' - \vec{q}} \hat{a}_{\vec{k}'} \hat{a}_{\vec{k}}$$