1) Using the Lehmann-spectral representation derive the expression for one-electron Green Function for the single interacting atom with the Hamiltonian:

$$\hat{H} = \sum_\sigma \varepsilon \hat{c}_\sigma^+ \hat{c}_\sigma + \frac{1}{2} \sum_{\sigma, \sigma'} U \hat{c}_\sigma^+ \hat{c}_{\sigma'}^+ \hat{c}_{\sigma'} \hat{c}_\sigma$$

where $\sigma = (\uparrow, \downarrow)$ are the spin projections.

2) Using the spin-polarized one-electron Green function:

$$G_\sigma(\vec{k}, i\omega) = \frac{1}{i\omega + \mu - \varepsilon_\sigma(\vec{k})}$$

obtain an expression for a magnetic susceptibility:

$$\chi_{\uparrow \downarrow}(q) = -\sum_k G_{\uparrow}(k)G_{\downarrow}(k + q)$$

where the 4-dimensional vector defined as:

$$k = (\vec{k}, i\omega)$$

$$\sum_k = T \sum_{\omega_n} \int \frac{d\vec{k}}{(2\pi)^3}$$